# Reexamination of scaling in the five-dimensional Ising model

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Abstract:

In three dimensions, or more generally, below the upper critical dimension, scaling laws for critical phenomena seem well understood, for both infinite and for finite systems. Above the upper critical dimension of four, finite-size scaling is more difficult.

Chen and Dohm predicted deviation in the universality of the Binder cumulants for three dimensions and more for the Ising model. This deviation occurs if the critical point $T = T_c$ is approached along lines of constant $A = L^2(T - T_c)/T_c$, then different exponents a function of system size $L$ are found depending on whether this constant $A$ is taken as positive, zero, or negative. This effect was confirmed by Monte Carlo simulations with Glauber and Creutz kinetics. Because of the importance of this effect and the unclear situation in the analogous percolation problem, we here reexamine the five-dimensional Glauber kinetics.

## 1. **Introduction**

In recent years, there is the question of universality of the five-dimensional Ising model. This question focuses on the value of susceptibility varying with temperatures near the critical temperature for different sizes of lattices, and here we investigate the susceptibility of the five-dimensional Ising model. In 2004 Chen and Dohm predicted theoretically [1], and then this prediction was partially confirmed [2-3], that the widely believed universality principle is violated in the Ising model on the simple cubic lattice with more than only six nearest neighbors. Other research groups [4-7] studied the 2D and 3D Ising model for different parameters and also for directed interactions problems occur in the Ising model. Schulte and Drope [3] by Monte Carlo

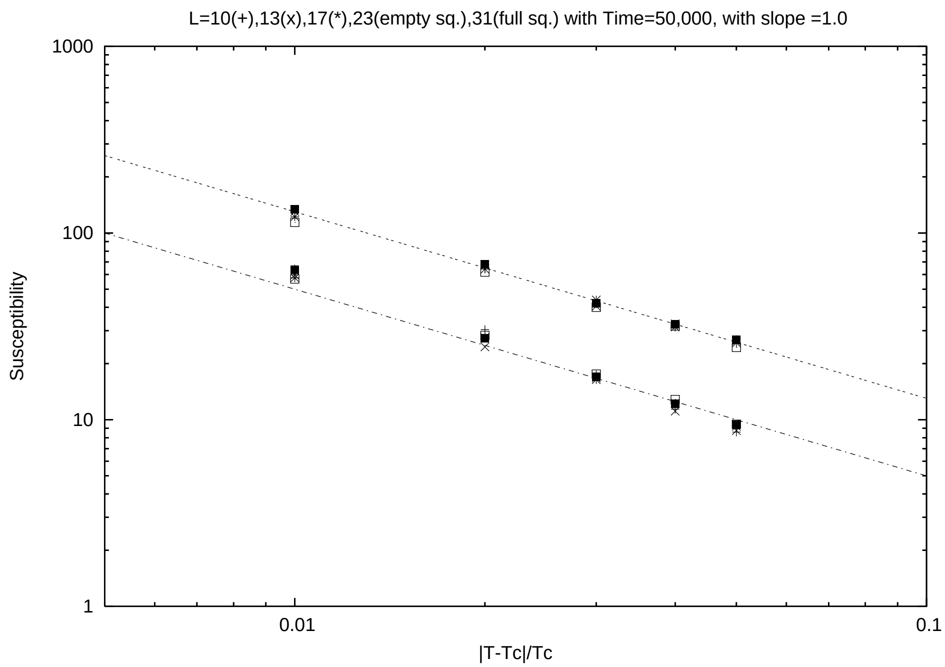


Figure 1: Susceptibility versus temperature with different $L$ (=10, 13, 17, 23, 31), for 10 nearest neighbors as log-log plot, the upper data correspond to $T > T_c$ with amplitude 1.3 , and the lower to $T < T_c$ with amplitude 0.5 , and straight lines had the theoretical slope (-1).

simulations with Glauber [8] and Creutz [9] kinetics, found such violation, but not in the predicted direction. Selke and Shchur [2] tested the square lattice, for this importance effect and the unclear situation in the analogous percolation [10], here we reexamine this universality for the susceptibility ratio and magnetization near the critical point. For this purpose we study first the standard 5D Ising model with ten nearest neighbors. Our study is based on Monte Carlo simulations for systems with linear different sizes (10, 13, 17, 31, 37, and 71). For the critical point: $J/kT_c = K_c = 0.113915$ is used as in ref. [11]. The FORTRAN program used for simulations is listed below.

2. **Problem**

Our problem here is to reexamine the universality scaling of 5D Ising model with the susceptibility and the magnetization along lines of constant

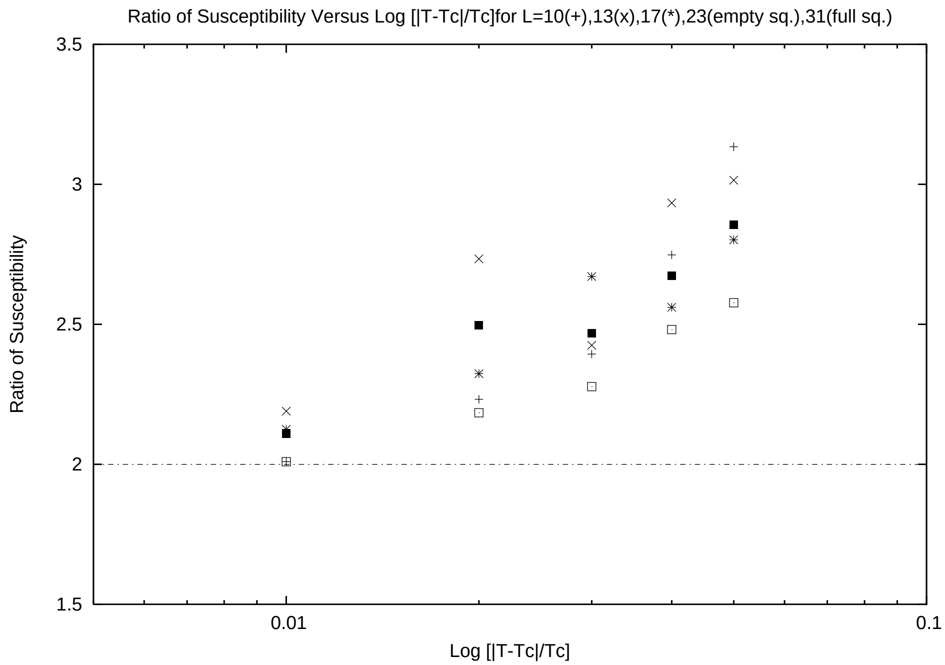


Figure 2: Ratio of susceptibility above to below $T_c$, plotted semi-logarithmically versus $|T_c - T|/T_c$, for $L$ = 10,13,17,23,31, for 10 neighbors up to time = 50000.

$A = L^2(T - T_c)/T_c$.

3. **Simulations and Results**

From our simulation for different sizes of lattice, by varying the temperature near the critical temperature we get the ratio of susceptibility by dividing the susceptibility of temperature above $T_c$ to the susceptibility below $T_c$ at the same $|T - T_c|$. Then the ratio of susceptibility was drawn versus $|T_c - T|/T_c$ as shown in figure (2) .

It can be seen that the ratio of susceptibility is roughly constant for varying size of lattice but increases away from the critical temperature. When large lattice as $L$ = 71 is tested for different times (500 and 5000), our simulation give the data as presented in figure 3.

This figure shows that the susceptibilities scatter much more than the magnetizations. Now we test the universality of 5D Ising model and vary $T$ along lines of constant $A = L^2(T - T_c)/T_c$ below, at and above $T_c$ with dif-

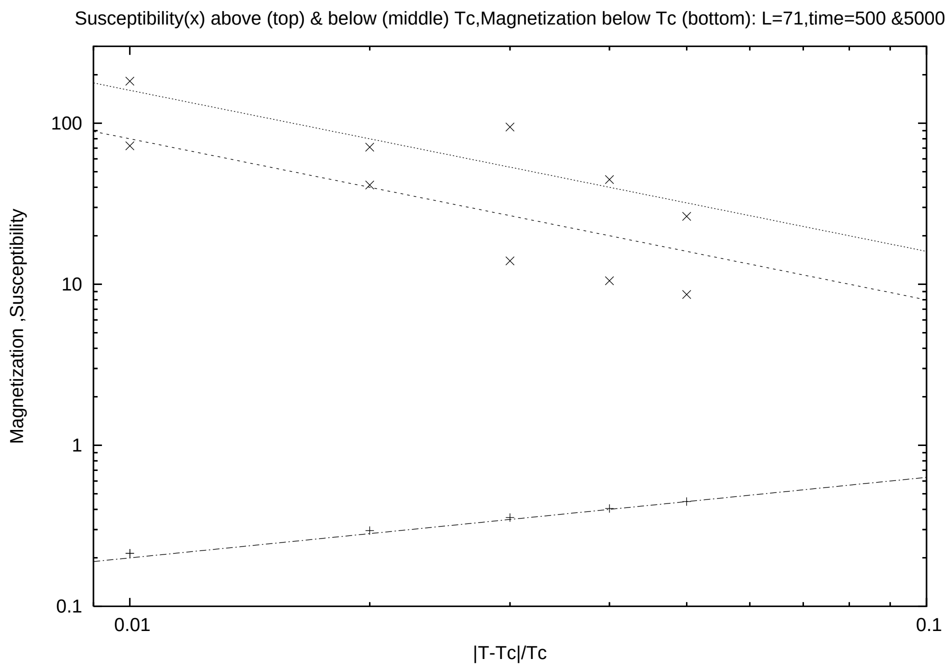


Figure 3: $|M|$ and susceptibility versus $|T_c - T|/T_c$ with fixed size L = 71 of lattice in log-log plot with lines indicating the theoretical slopes -1 and + .

ferent $L$ (10,13,17,23,31) for long time (500000). Then the data are obtained as seen in figures 4 and 5.

Now if the average of the absolute value of magnetization is taken, and plotted with the size of lattices with all constants $A = +1, 0, -1$ with log-log plot , the slopes are obtained in figure 4, in agreement with previous theories and simulations [1,8,9].

Then by drawing the susceptibility versus the size $L$ of lattices for the constants $A = +1, 0, -1$ with log-log plot , we get different slopes, twice as large as for the magnetization in the previous figure, as shown in figure 5.

4. **Programming used in Simulations:**

A: Main program:

```
   PARAMETER(L=17,L2=L*L,L3=L2*L,L4=L2*L2,L5=L3*L2,
  1 LMAX=L5+2*L4)
   INTEGER *8 IBM,IEX
```

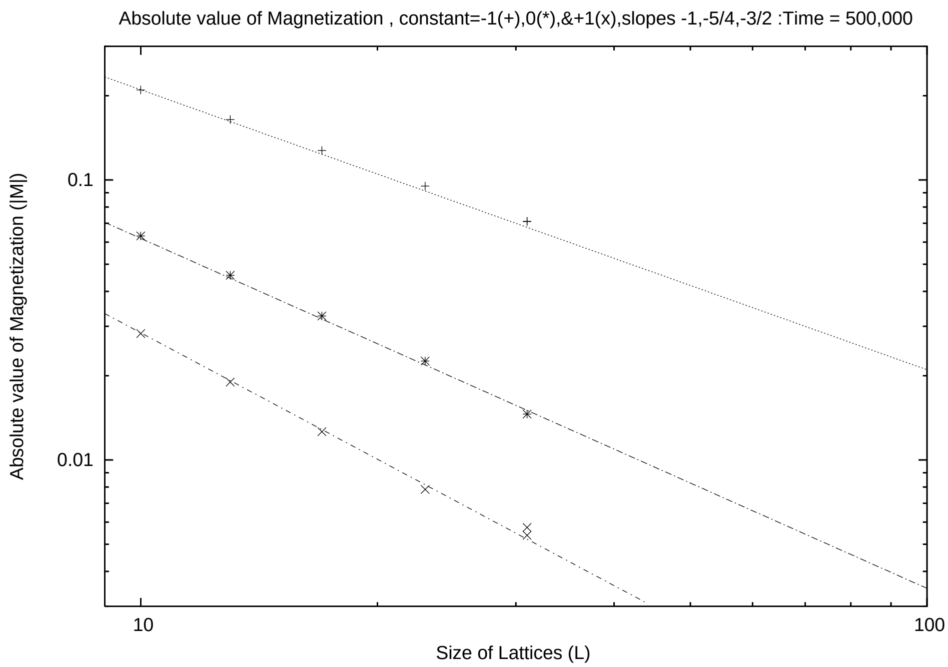


Figure 4: $< |M| >$ versus size $L$ of lattice (10, 13, 17, 23, 31), in log-log plot along constant $A = L^2(T - T_c)/T_c$. The upper data correspond to $T < T_c$ ($A = -1$) with slope –1, the middle data to $T = T_c$ ($A = 0$) with slope –5/4, and the lower to $T > T_c$ ($A = +1$) with slope –3/2.

```
      DIMENSION IEX(-10:10)
      BYTE IS(LMAX)
      DATA TC,MAX,IBM,ISEED/0.113915,500000,1,1/
      IBM=2*ISEED -1
      CONST=0.0
      T=-(TC*CONST/L2)+TC
      T1=TC/T-CONST
C     T= TI*(1.0-0.1/(L*L))
      PRINT *,L,T,T1,MAX,ISEED
      LP1=L4+1
      L2PL=L5+L4
      DO 1 I=1,LMAX
1     IS(I)=1
      DO 2 IE=-10,10
```

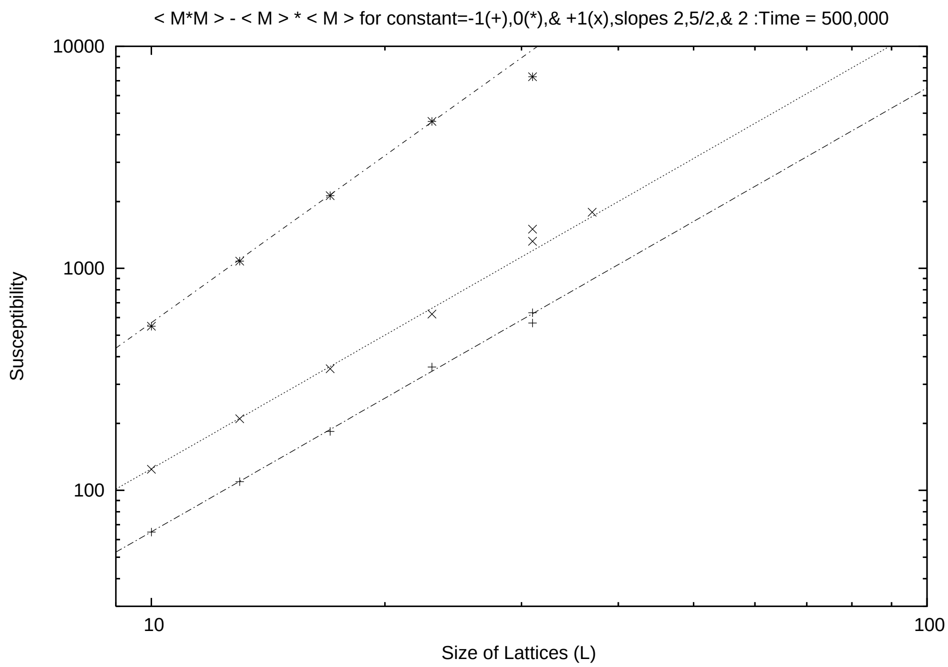


Figure 5: Susceptibility $< M^2 > - < M >^2$ versus size $L$ of lattice (10, 13, 17, 23, 31, (37)), as log-log plot along constant $A = L^2(T - T_c)/T_c$, the upper data correspond to $T = T_c$ ($A = 0$), the middle data to $T > T_c$ ($A = 1.0$), and the lower data to $T < T_c$ ($A = -1$). The middle data fit better the indicated slope 2 than the expected slope 3.

```
      IBM=IBM*16807
      EX=EXP(-2.0*IE*T)
2     IEX(IE)=2147483648.0D0*(4.*EX/(1.0+EX)-2.0)*2147483648.0D0
      DO 3 MC=1,MAX
      DO 4 I=LP1,L2PL
      IE=IS(I)*(IS(I-1)+IS(I+1)+IS(I-L)+IS(I+L)+IS(I-L2)+IS(I+L2)
     1   +IS(I-L3)+IS(I+L3)+IS(I-L4)+IS(I+L4))
      IBM=IBM*16807
      IF (IBM.LT.IEX(IE)) IS(I)= -IS(I)
      IF(I.NE.2*(L4)+1) GOTO 4
      DO 7 J=1,L4
7     IS(J+L5+L4)=IS(J+L4)
4     CONTINUE
```

```
      FACTOR=1.0/(L*L*L*L*L)
      DO 5 I=1,L4
5     IS(I)=IS(I+L5)
      MAGN=0
      DO 6 I=LP1,L2PL
6     MAGN=MAGN+IS(I)
      X=MAGN*FACTOR
3     PRINT *,MC,MAGN,X
      STOP
      END
```

B: Analysis program:

```
      INTEGER*8 MAGN,SUMMAG,SUMSQU
      REAL*8 X, AVERGESUMMAG,AVERGESUMSQU
      READ *,L,T,T1,MAX,ISEED
      L5=L*L*L*L*L
      SUMMAG=0
      SUMSQU=0
      ISUMMAG=0
      DO 100 I=1,MAX
      READ *,MC,MAGN
      X=MAGN
      IF(MC.LE.(MAX/2)) GO TO 100
      SUMMAG=SUMMAG+X
      ISUMMAG=ISUMMAG+MAGN
      SUMSQU=SUMSQU+X*X
C     PRINT *, MC, ISUMMAG,ISUMSQU
100   CONTINUE
      AVERGESUMMAG=SUMMAG/(MAX*0.5D0)
      AVERGESUMSQU=SUMSQU/(MAX*0.5D0)
      X=AVERGESUMMAG/L5
      CHI=(AVERGESUMSQU-AVERGESUMMAG**2)/L5
      PRINT 1,L,T,X,CHI,ABS(T),X*X*CHI
1     FORMAT (1X,I2,5F15.5)
      STOP
      END
```

## 5. Conclusion

We thus confirmed [1,8,9] that finite size scaling in high dimensions is described by different exponents if we approach the critical point along different lines in the plane of $T - T_c$ versus $1/L^2$, above, at and below $T_c$. This holds is not only for the magnetization [8] but also for the susceptibility, though the susceptibilities above $T_c$ are problematic.

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